X klt cpt Kähler, $T \subset \mathrm{Aut}_r(X)$.
$Y \to X$ T-equiv. resolution
$\Omega_X = (\omega_X, m_X)$
ν_X : adapted measure $\quad v, w : t^\vee \to \mathbb{R}$
positive on $P_X := m_X(X)$.

$$M_X^{rel} = M^{rel}_{\Omega_{X,v,w}} : \mathcal{E}_X^{1,T} \to \mathbb{R} \cup \{+\infty\}.$$

$$M_X^{rel} = H_{X,v}(\psi) + R_{X,v}(\psi) + E_{X,vw\ell_X}(\psi).$$

$$H_{X,v}(\psi) := \frac{1}{2} \mathrm{Ent}(MA_{X,v}(\psi) | \nu_X)$$

$$R_{X,v}(\psi) := E^{-\mathrm{Ric}^T(\nu_X)}_{\Omega_{X,v}}(\psi).$$ (ν_X comes with a C^∞ metric on K_X).

$$E_{X,vw\ell_X}(\psi) = E_{\Omega_{X,vw\ell_X}}(\psi) \qquad \ell_X := \ell^{ext}_{\Omega_{X,v,w}}.$$

$$\int_X \ell(m_\Omega)\, \ell^{ext}(m_\Omega)\, w(m_\Omega)\, MA_v(0) = \int_X \ell(m_\Omega)\, S_v(\Omega)\, MA_v(0).$$

ν_Y on Y, $\Omega_j = (\omega_j, m_j)$ on Y. equiv. Kähler form.

1) $\Omega_j \to \pi^*\Omega_X$ smoothly
2) $\omega_j \geq (1-\varepsilon_j)\pi^*\omega_X$ $\quad [0,1) \ni \varepsilon_j \to 0$

Ex. X smooth, $Y = \mathrm{Bl}_p X$, E exceptional.
$\Theta_E = (\theta_E, m_E)$ form.
s.t. $\omega_Y := \pi^*\omega_X - \varepsilon\theta_E > 0$ for $\alpha\varepsilon \ll 1$
$\omega_j = (1 - \varepsilon_j/\varepsilon)\omega_X + (\varepsilon_j/\varepsilon)\omega_Y \to \omega_X$.

$$M_j^{rel}(\varphi) = H_{j,v}(\varphi) + R_{j,v}(\varphi) + E_{j,vw\ell_j}(\varphi).$$

Def. $\pi: Y \to X$ Fano type if $\exists$ q-psh f on Y s.t.

- $\hat{\nu}_Y := e^{-2f}\nu_Y$ finite measure.
- $\mathrm{Ric}(\hat{\nu}_Y) := \mathrm{Ric}(\nu_Y) + dd^c f$ is π-semipositive

$\exists$ $(1,1)$-form α on X s.t. $\pi^*\alpha + \mathrm{Ric}(\hat{\nu}_Y) \geq 0$.

don't have to ~~state~~ mention this.

Ex. X smooth, $Y = \mathrm{Bl}_p X$. $-K_Y$ is π-ample

$\exists$ metric on $-K_Y$ $\longleftrightarrow$ $\hat{\nu}_Y = e^{-2f}\nu_Y$, $f \in C^\infty(Y)$ s.t. $\mathrm{Ric}(\hat{\nu}_Y)$ is π-semiample

Lem. X, Y proj. $\pi: Y \to X$ of Fano type iff $\exists$ effective $\mathbb{Q}$-divisor B on Y s.t.

(i) $-(K_Y + B)$ is π-ample.

(ii) (Y, B) klt.

$K_Y = \pi^* K_X + D$. $\quad dd^c \phi_D = 2\pi[D]$ $\longleftarrow$ ϕ_D metric on $\mathcal{O}_Y(D)$

$\phi_{Y/X} := \frac{1}{2}\log \nu_Y - \pi^* \frac{1}{2}\log \nu_X$ $\longleftarrow$ metric on $\mathcal{O}_Y(D)$.

$\pi^*\nu_X = e^{2\rho_{Y/X}}\nu_X$ $\quad e^{2\rho_{Y/X}} \in L^p(Y)$, $p > 1$.

$$e^{2\rho_{Y/X}} = \frac{\prod |s_i|^{2a_i}}{\prod |t_j|^{2b_j}}$$

$\rho_{Y/X} = \phi_D - \phi_{Y/X}$.

After scaling ν_Y, $\int_Y \rho_{Y/X} MA_{Y,\nu}(0) = 0$.

$\Theta_{Y/X} := dd^c_T \phi_{Y/X}$.

$\Theta_{Y/X} + dd^c_T \rho_{Y/X} = dd^c_T \phi_D = 2\pi[D]_T$

Utiliser cette page pour les solutions, les développements et les réponses.

Colonne réservée à la correction

For $\forall \psi \in \mathcal{E}^{1,T}_X$,

$$H_{Y,v}(\pi^*\psi) := \frac{1}{2} \mathrm{Ent}\left(MA_{Y,v}(\pi^*\psi) \mid \nu_Y\right)$$

if ψ smooth, $R_{Y,v}(\pi^*\psi) := E^{-\mathrm{Ric}^T(\nu_Y)}_{\pi^*\Omega_{X,v}}(\pi^*\psi)$.

$\Rightarrow$ Rk. $R_{Y,v}$ cannot a priori be extended to $\pi^*\mathcal{E}^{1,T}_X$

~~Rk. $R_{Y,v}$ cannot a priori be extended to $\pi^*\mathcal{E}^{1,T}_X$~~

Rk. $R_{X,v}$ can extend to $\mathcal{E}^{1,T}_X$ since ω_X is Kähler.

Lem. $\forall \psi \in \mathcal{H}^T_X$, $H_{X,v}(\psi) + R_{X,v}(\psi) = H_{Y,v}(\pi^*\psi) + R_{Y,v}(\pi^*\psi)$.

$$\Theta_{Y/X} = \pi^*\mathrm{Ric}^T(\nu_X) - \mathrm{Ric}^T(\nu_Y)$$

$$\Rightarrow R_{Y,v}(\pi^*\psi) = R_{X,v}(\psi) + E^{\Theta_{Y/X}}_{\pi^*\Omega_{X,v}}(\pi^*\psi)$$

Assume

$$MA_{X,v}(\psi) = f^2 \nu_X \qquad f \in C^\infty(X, \mathbb{R}_{>0}).$$

$$H_{X,v}(\psi) = \int_X \log f \, MA_{X,v}(\psi)$$

$$MA_{Y,v}(\pi^*\psi) = \pi^* f^2 \cdot \pi^*\nu_X = \pi^* f^2 \cdot e^{2\rho_{Y/X}} \nu_Y.$$

$$H_{Y,v}(\pi^*\psi) = \frac{1}{2}\int_Y \log\left(\frac{MA_{Y,v}(\pi^*\psi)}{\nu_Y}\right) MA_{Y,v}(\pi^*\psi)$$

$$= H_{X,v}(\psi) + \int_Y \rho_{Y/X} \, MA_{Y,v}(\pi^*\psi).$$

$\varphi \mapsto \int_X g \, MA_{\Omega,v}(\varphi)$ is an E-L func. for $MA^{dd^c_T g}_{\Omega,v}(\varphi)$

$$\left(H_{Y,v}(\pi^*\psi) + R_{Y,v}(\pi^*\psi)\right) - \left(H_{X,v}(\psi) + R_{X,v}(\psi)\right)$$

$$= E^{\Theta_{Y/X}}_{\pi^*\Omega_{X,v}}(\pi^*\psi) + \int_Y \rho_{Y/X} \, MA_{Y,v}(\pi^*\psi).$$

$$\boxed{\begin{array}{l}\text{Lem. For any } T\text{-inv. distribution } g \text{ and } \varphi, \psi \in C^\infty(X)^T, \\ \text{we have } E^{dd^c_T g}_{\Omega,v}(\varphi) - E^{dd^c_T g}_{\Omega,v}(\psi) = \int_X g \left(MA_{\Omega,v}(\varphi) - MA_{\Omega,v}(\psi)\right)\end{array}}$$

$$= E^{\Theta_{Y/X} + dd^c_T \rho_{Y/X}}_{\pi^*\Omega_{X,v}}(\pi^*\psi) = 2\pi E^{[D]_T}_{\pi^*\Omega_{X,v}}(\pi^*\psi) = 0.$$ D is exceptional

Utiliser cette page pour les solutions, les développements et les réponses.

Colonne réservée à la correction

prop. For any sequence $\varphi_j \in \mathcal{H}_j^T$ converging C^∞ to $\pi^*\varphi$ with $\varphi \in \mathcal{H}_X^T$, we have

$$H_{j,v}(\varphi_j)+R_{j,v}(\varphi_j)+E_{j,w\ell_j}(\varphi_j) = M_j^{rel}(\varphi_j) \to M_X^{rel}(\varphi). = \underbrace{H_{X,v}(\varphi)+R_{X,v}(\varphi)}_{\shortparallel}+E_{X,w\ell_X}(\varphi)$$

$$H_{Y,v}(\pi^*\varphi)+R_{Y,v}(\pi^*\varphi).$$

Lem. ℓ_j converges to ℓ_X in $\mathfrak{t}\oplus\mathbb{R}$.

$$\int_X \ell(m_X)\,\ell_X(m_X)\,w(m_X)\,MA_{X,v}(0) = \int_X \ell(m_X)\,S_v(\Omega_X)\,MA_{X,v}(0).$$

$\uparrow$ ~~ clear not clear. ~> $\uparrow$

$$\int_Y \ell(m_j)\,\ell_j(m_j)\,MA_{Y,vw}(0) = \int_Y \ell(m_j)\,S_v(\Omega_j)\,MA_{Y,v}(0)$$

Lem. $f_j \in C^\infty(Y)$ converges smoothly to π^*f with $f \in C^\infty(X)$. Then.

$$\int_Y f_j S_v(\Omega_j)\,MA_{j,v}(0) \to \int_X f S_v(\Omega_X)\,MA_{X,v}(0).$$

Pf. $\rho_X := \frac{1}{2}\log\left(\frac{MA_{X,v}(0)}{\nu_X}\right)$. $\rho_j := \frac{1}{2}\log\left(\frac{MA_{j,v}(0)}{\nu_Y}\right)$.

qpsh on X.

$$f S_v(\Omega_X)MA_{X,v}(0) = f MA_{X,v}^{Ric_v^T(\Omega_X)}(0)$$
$$= f MA_{X,v}^{Ric^T(\nu_X)}(0) - f MA_{X,v}^{dd_T^c\rho_X}(0)$$

$$f_j S_v(\Omega_j)MA_{j,v}(0) = f_j MA_{j,v}^{Ric^T(\nu_Y)}(0) - f_j MA_{j,v}^{dd_T^c\rho_j}(0)$$
$$= f_j MA_{j,v}^{\pi^*Ric^T(\nu_X)}(0) - f_j MA_{j,v}^{\Theta_{Y/X}+dd_T^c\rho_j}(0)$$

$$\rho_j \longrightarrow \pi^*\rho_X + \rho_{Y/X} \quad \text{in } L^1.$$

$$\int_Y f_j MA_{j,v}^{dd_T^c\rho_j}(0) \overset{\text{Sym.}}{=} \int_Y \rho_j MA_{j,v}^{dd_T^c f_j}(0) \longrightarrow \int_Y (\pi^*\rho_X+\rho_{Y/X})MA_{\pi^*\Omega_X,v}^{dd_T^c\pi^*f}(0)$$
$$= \int_X f MA_{X,v}^{dd_T^c\rho_X}(0) + \int_Y \pi^*f MA_{\pi^*\Omega_X,v}^{dd_T^c\rho_{Y/X}}$$

$$\int_Y f_j MA_{j,v}^{\Theta_{Y/X}+dd_T^c\rho_j}(0) \to \int_X f MA_{X,v}^{dd_T^c\rho_X}(0) + \int_Y \pi^*f\, MA_{\pi^*\Omega_X,v}^{dd_T^c\rho_{Y/X}+\Theta_{Y/X}}(0)$$

$= D$ D exceptional

Lem. Let $\Theta = (\theta, m_\theta)$ be an equiv. current, Θ: π-semipositive $\left(\exists \text{ equiv. form } \Theta \text{ s.t. } \pm\Theta \le C\pi^* \omega_X\right)$. Then

i). $E^\theta_{j,v} : \mathcal{H}^T_j \to \mathbb{R}$ and $E^\theta_{Y,v} : \pi^* \mathcal{H}^T_X \to \mathbb{R}$ admit unique usc (continuous) extensions.

$$E^\theta_{j,v} : \Sigma^{1,T}_j \to \mathbb{R} \cup \{-\infty\} \quad E^\theta_{Y,v} : \pi^* \Sigma^{1,T}_X \to \mathbb{R} \cup \{-\infty\}$$

that are continuous along decreasing sequences.

ii) for any strongly convergent sequence $\Sigma^{1,T}_j \ni \varphi_j \to \pi^* \psi$ with $\psi \in \Sigma^{1,T}_X$, we have

$$\limsup_j E^\theta_{j,v}(\varphi_j) \le E^\theta_{Y,v}(\pi^*\psi)$$
$$(\lim) \quad (=)$$

iii) $\exists A > 0$ s.t.

$$|E^\theta_{j,v}(\varphi)| \le A(d_{1,j}(\varphi, 0) + 1)$$

for all j and all $\varphi \in \Sigma^{1,T}_j$.

decreasing sequence in Σ^1 $\varphi_j \to \varphi$ is convergent

Pf. May assume $\theta \ge 0$ (pullback Ω_X)

$$\theta' = \theta - dd^c_T f \quad \text{smooth} \quad f \ \text{~~psh~~ } \theta\text{-psh } T\text{-inv.}$$

$E^{\theta'}_{j,v}$ $\left(E^{dd^c_T f}_{j,v}\right)$ admit continuous (usc) extension.

$F_j = E^\theta_{j,v} + A E_{j,v}$ monotone increasing for $A > 0$ ind. of j.

~~Cheese~~

Lem. equiv. form $\Theta = (\theta, m_\theta)$, $\exists$ cpt $K \subset t^\vee$ only depending on θ s.t. the following holds: if $v, w \in C^\infty(t^\vee)$ satisfy:

$$v(\alpha) \ge 0, \quad \langle v'(\alpha), \beta \rangle + w(\alpha) \ge 0 \quad \forall \alpha \in P, \beta \in K.$$

Then $E^{\Theta + dd^c_T f}_v + E_w$ is monotone increasing for $\forall$ θ-psh f on $\mathcal{H}^T$.

$F_j : \mathcal{F}_j \to \mathbb{R} \cup \{-\infty\}$, $\mathcal{F}_j \subset \Sigma_j^!$

Assume $\mathcal{R} \subset \mathcal{F} \subset \Sigma'$ and $F : \mathcal{R} \to \mathbb{R}$. s.t.

(R) $\mathcal{R}$ consists of bounded function. each $\varphi \in \mathcal{F}$ is limit of some decreasing φ^k in $\mathcal{R}$

(F) Each $\varphi \in \mathcal{R}$, $(1-\varepsilon_j)\varphi$ lies in $\mathcal{F}_j$ and

$F_j((1-\varepsilon_j)\varphi) \to F(\varphi)$.

Lem. 2.13 | Assume F_j is finite valued and satisfies

(1) $|F_j(\varphi) - F_j(\psi)| \le C\, d_{1,j}(\varphi,\psi)^\alpha \max\{d_{1,j}(\varphi,0), d_{1,j}(\psi,0)\}^{1-\alpha}$

for all $\varphi, \psi \in \mathcal{F}_j$, $\alpha \in (0,1]$ and $C>0$ are uniform constants. Then F admits a unique continuous extension $F : \mathcal{F} \to \mathbb{R}$ which also satisfies (1). For each strongly convergent $\mathcal{F}_j \ni \varphi_j \to \varphi \in \mathcal{F}$, we have $F_j(\varphi_j) \to F(\varphi)$.

Lem 2.14 |

Assume i) $\mathcal{R} \subset C^0$, $\mathcal{R}$ contain constants

ii) Each F_j is translation inv, F_j is monotone increasing and $F_j(\varphi + c) = F_j(\varphi) + c a_j$ ← translation-equiv

Then F admits a unique extension $F : \mathcal{F} \to \mathbb{R} \cup \{-\infty\}$ continuous along decreasing sequences. F is weakly usc. monotone increasing and translation equiv. and for any weakly convergent seq $\mathcal{F}_j \ni \varphi_j \to \varphi \in \mathcal{F}$, we have

$$\limsup_j F_j(\varphi_j) \le F(\varphi).$$

Choose $A > 0$ s.t.

$\langle v(\alpha), \beta \rangle + A v(\alpha) \geq 0$ for all $\alpha \in P_j$, $\beta \in K$.

Apply Lemma to θ' $\Rightarrow$ $E_{j,v}^{\theta' + dd^c_T f} + A E_{j,v}$ monotone increasing.

Since $\theta' + dd^c f = \theta \geq 0$.

$\Rightarrow$ F_j monotone increasing on $\mathcal{H}_j^T$. ($\mathcal{E}_j^{1,T}$).

Need to check (F) also. For $\psi \in \mathcal{H}_X^T$, $(1-\varepsilon_j)\pi^*\psi \xrightarrow{C^\infty} \pi^*\psi$ implies

$E_{j,v}^{\theta}((1-\varepsilon_j)\pi^*\psi) \to E_{Y,v}^{\theta}(\pi^*\psi)$ $\quad E_{j,v}((1-\varepsilon_j)\pi^*\psi) \to E_{Y,v}(\pi^*\psi)$.

Lem 2.14 to $F = E_{Y,v}^{\theta} + A E_{Y,v} : \pi^*\mathcal{H}_X^T \to \mathbb{R}$

$\Rightarrow$ $F : \pi^*\mathcal{E}_X^{1,T} \to \mathbb{R} \cup \{-\infty\}$ continuous along decreasing sequences, weakly usc, monotone increasing translation equiv. s.t.

$$\limsup_j F_j(\varphi_j) \leq F(\varphi)$$

for $\forall$ weakly convergent $\mathcal{E}_j^{1,T} \ni \varphi_j \to \varphi \in \pi^*\mathcal{E}_X^{1,T}$

$E_{Y,v}$ is strongly continuous on $\pi^*\mathcal{E}_X^{1,T}$ $\Rightarrow$ $E_{Y,v}^{\theta}$ satisfies (i), (ii).